Analysis on Feedback Interconnections of Cascaded DC-DC Converter Systems

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***Abstract*—Stability analysis of cascaded DC-DC converters systems is one of fundamental issues crucial to real-world power supplies applications. Existing techniques for stability analysis of cascaded converters systems are mainly based on Thevenin or Norton equivalents of two-port networks of sub-converters in which impedance-based stability criteria can be applied. In this work, further transformations of Thevenin or Norton equivalents to feedback interconnections of two general classes, i.e. heterogeneous and homogeneous cascaded converter systems are presented to facilitate simpler stability evaluation via the small gain theorem. Qualitative analysis on feedback interconnections is then conducted to reveal the differences in using the impedance-based stability criteria of cascaded converter systems. Finally, a laboratory prototype of heterogeneous cascaded DC-DC converter system is built to verify the analysis.**

Keywords—Cascaded systems; DC-DC converter; stability; small-gain theorem; feedback systems

I. INTRODUCTION

In distributed power supplies, cascaded DC-DC switching converters are very common structures to power various point-of-loads (POLs). Stability analysis of cascaded DC-DC converter systems is one of fundamental issues crucial to real-world applications because of the interactions between the upstream and downstream converters [1, 2]. Historically, the studies on stability of cascaded switching converters can be dated back to the input filter designs of the switching regulators in 1970s. Often, the Buck-type switching regulators were equipped with input filters in order to reduce the input current ripple. Unfortunately, the additional input filter would lead to the change of the original control loop gain of the switching regulator and finally the control performance degradation. To circumvent this problem, R. D. Middlebrook proposed the design rule for the input filter that the ratio of the output impedance of the input filter to the input impedance of the switching regulator should be far smaller than 1 to decouple the interaction between the input filter and the regulator, which is then known as Middlebrook's impedance criterion and then extended to the general cascaded converters systems today. As shown in Fig. 1, the impedance ratio (Z_{OS}/Z_{iL}) is also defined as the minor loop gain (MLG) to determine and measure the stability of the overall cascaded converter system, where the Z_{OS} is the output impedance of the source converter and the Z_{iL} the input impedance of the load converter [3]. However, it was found that Middlebrook's impedance criterion was too conservative to be satisfied in practical designs. In the following decades, several similar impedance-based criteria have been proposed for the purposes of reducing the conservatism of the impedance-based stability criteria and optimizing stability monitoring and control parameters [4-7].

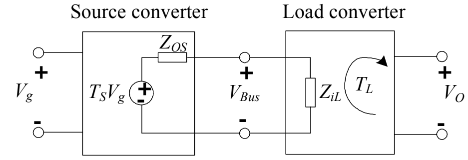

Fig. 1 Typical Cascaded Converter system

On the other hand, various kinds of cascaded converters systems began to appear with the fast growing renewable energy applications in which different control characteristics of sub-converters in their cascaded connection may produce different impedance-based stability criteria [8, 9]. It has been proven that the MLG for the grid-connected inverter system is that inverse of the output voltage regulated sub-converters in cascading, where the source converter is of current source in the former application and of voltage source in the latter application[8]. Also, the stability of cascaded system for both source converter and load converter are of voltage source was analyzed based on another different impedance criterion [10]. It also should be noted that most of the existing analyses are derived from the two-port network models, i.e. Norton equivalent circuits for current sources and Thevenin equivalent circuits for voltage sources of the input ports or output ports of the sub-converters in cascaded systems [4, 11].

In this paper, analysis for general cascaded DC-DC converter systems is conducted on equivalent transformation of two-port network models into their feedback interconnections. By classification of four kinds of generally cascaded converter systems into heterogeneous and homogeneous ones, the differences in using the impedance-based stability criteria are easily revealed via small gain theorem. The remaining of the paper is organized as follows. In Section II, feedback interconnections for both the heterogeneous and homogeneous cascaded systems are derived based on the equivalent circuit models. The stability

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analysis is then conducted in Section III. Experiments for one kind of heterogeneous cascaded system are given in Section IV. The conclusion is drawn finally in Section V.

II. Equivalent Modeling by Feedback Interconnections

For a DC-DC switching converter, the behavior model by using the linear two-port network will be able to describe its input and output properties [11]. In cascaded connections of switching converters systems, there will be four combinations out of two-port properties of sub-converters (source and load converter) by using either Thevenin or Norton equivalent circuits for the input or output ports. Here, we classify the two in Fig. 2 are heterogeneous cascaded converter systems where the source converter and load converter are of different source types, i.e. one sub-converter is expressed in voltage source and the other in current source. It can also be noticed that the two kinds of cascaded converter systems in Fig. 2(a) and (b) are dual to each other. Similarly, the homogeneous cascaded converter systems are shown in Fig. 3 where both the source converter and load converter are expressed either all voltage sources or all current sources. Also, the cascaded converter systems in Fig. 3(b) can be derived from Fig. 3(a) by duality theorem. Another reason for this classification is that the feedback interconnections would be uniquely determined in each heterogeneous cascaded converter system, so would be the minor loop gain, which is not true in homogeneous cascaded systems. In the following, the equivalent models by the feedback interconnections will be derived based on the circuit models.

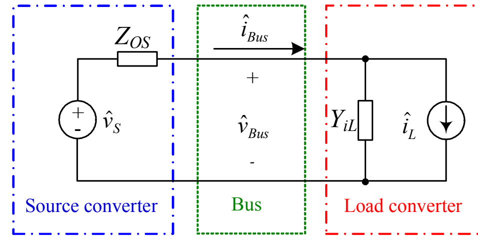
To derive the feedback interconnections equivalents, we see the heterogeneous cascaded converter system in Fig. 2(a) first. The circuit equations of Fig. 2(a) are found by KCL (Kirchhoff's Current Law) and KVL (Kirchhoff's Voltage Law) to be

$$\begin{cases} \hat{i}_{Bus} = \hat{i}_L + Y_{iL}\hat{v}_{Bus} \\ \hat{v}_{Bus} = \hat{v}_S - Z_{OS}\hat{i}_{Bus} \end{cases} \tag{1}$$

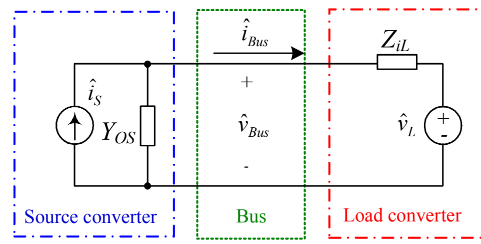
where $\hat{i}_{Bus}$ and $\hat{v}_{Bus}$ are the bus current and bus voltage respectively, and $\hat{i}_L$ and $\hat{v}_S$ are the load current and source voltage respectively, and Y_{iL} and Z_{OS} are input admittance of the load converter and output impedance of the source converter respectively.

According to (1), the equivalent feedback interconnection of Fig. 2(a) is uniquely derived as Fig. 4(a), which also means that the feedback interconnection model is totally same as Fig. 2 (a) in mathematical sense.

Similarly, we can also uniquely determine the feedback interconnection of Fig. 2(b) as Fig. 4(b) from which the mathematical equations for bus current and bus voltage are given as

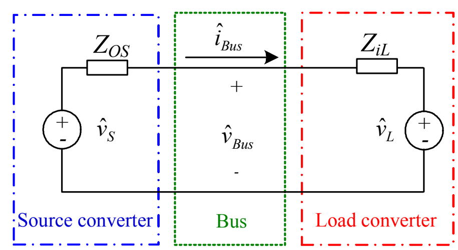


(a) Voltage type source converter cascaded by current type load converter

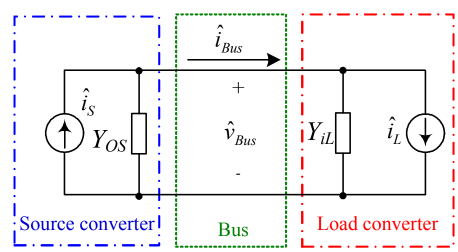


(b) Current type source converter cascaded by voltage type load converter

Fig. 2 Heterogeneous cascaded converter systems



(a) Both sub-converter are of voltage source types



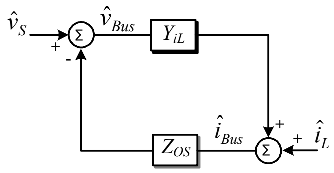
(b) Both sub-converter are of current source types

Fig. 3 Homogeneous cascaded converter systems

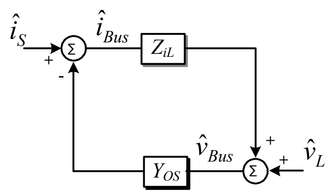
$$\begin{cases} \hat{i}_{Bus} = \hat{i}_S - Y_{OS}\hat{v}_{Bus} \\ \hat{v}_{Bus} = \hat{v}_L + Z_{iL}\hat{i}_{Bus} \end{cases} \tag{2}$$

The feedback loop structures in Fig. 4 have been chosen because that its symmetry is convenient for analysis and it can also remain structure invariance under some loop transformations [12].

However, in homogeneous cascaded converter systems as Fig. 3 (a) and (b), the feedback interconnection for each one can not be uniquely determined as that of heterogeneous cascaded converter systems.



(a) Feedback interconnection equivalent of Fig. 2(a)



(b) Feedback interconnection equivalent of Fig. 2(b)

Fig. 4 Unique Feedback interconnections for each heterogeneous cascaded converter system

As shown in Fig. 3 (a) first, the governing equations for this cascaded system can either be derived as

$$\begin{cases} \hat{i}_{Bus} = \left(\hat{v}_{Bus} - \hat{v}_L\right) / Z_{iL} \\ \hat{v}_{Bus} = \hat{v}_S - \hat{i}_{Bus} Z_{OS} \end{cases} \tag{3a}$$

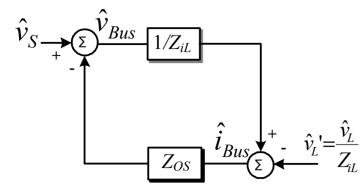
or be derived as

$$\begin{cases} \hat{i}_{Bus} = \left(\hat{v}_S - \hat{v}_{Bus}\right) / Z_{OS} \\ \hat{v}_{Bus} = \hat{v}_L + \hat{i}_{Bus} Z_{iL} \end{cases} \tag{3b}$$

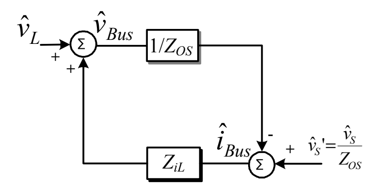
Based on (3a) and (3b), their equivalent feedback interconnections are obtained in Fig. 5(a) and (b) respectively. Both the feedback interconnections make sense even for the only one homogeneous cascaded converter system in Fig. 3(a), because we can only get one solution for the bus current and voltage as

$$\begin{cases} \hat{v}_{Bus} = \dfrac{\hat{v}_S Z_{iL} + \hat{v}_L Z_{OS}}{Z_{iL} + Z_{OS}} \\ \hat{i}_{Bus} = \dfrac{\hat{v}_S - \hat{v}_L}{Z_{iL} + Z_{OS}} \end{cases} \tag{4}$$

Derivation of feedback interconnection of the other homogeneous cascaded converter system in Fig. 3(b) is similar to that of Fig.3(a) and it will also lead to non-unique feedback interconnection and not be discussed hereafter. Therefore, the unique equivalent feedback interconnections for the heterogeneous cascaded converter systems in Fig.3 are totally different from those of homogeneous cascaded systems in Fig. 4 where the feedback interconnections cannot be uniquely determined. From this point of view, the analysis of the homogeneous cascaded converters systems may be ambiguous by existing techniques, especially in deriving the stability criteria due to the non-uniqueness of minor loop gains for homogeneous cascaded systems [4, 10].



(a) Feedback interconnection equivalent based on (3a)



(b) Feedback interconnection equivalent based on (3b)

Fig. 5 Non-unique feedback interconnections for only the homogeneous cascaded converter system in Fig. 3 (a)

III. Stability Criteria Based on Feedback Interconnections

Existing methods for stability analysis of cascaded converter systems are mainly based on the equivalent circuit models. For example, the bus current and voltage of cascaded system in Fig. 2 (a) can be solved as

$$\begin{cases} \hat{i}_{Bus} = \dfrac{\hat{i}_L + Y_{iL}\hat{v}_S}{1 + Y_{iL} Z_{OS}} \\ \hat{v}_{Bus} = \dfrac{\hat{v}_S - Z_{OS}\hat{i}_L}{1 + Y_{iL} Z_{OS}} \end{cases} \tag{5}$$

To satisfy the stability requirement, the denominator of (5) must not have any roots in the right half plane, from which one can define $Y_{iL}Z_{OS}$ the minor loop gain (MLG) by analogy.

Here, the stability criteria will be derived based on the feedback interconnections for each heterogeneous cascaded converter systems in Fig. 4 first. For the interconnected feedback system in Fig. 4(a), application of the well-known small-gain theorem [13] yields the sufficient stability condition for the heterogeneous cascaded converter system in Fig. 2(a) as

$$\left|Y_{iL}\cdot Z_{OS}\right|<1 \tag{6}$$

provided that both the sub-converters, i.e. voltage type source converter and current type load converter are stable in stand-alone fashion.

Equation (6) shows that the small gain theorem gives the stability condition in the sense magnitude of the output impedance of the source converter and the input admittance of the load converter, and is a simpler and more direct way to judge the stability of the cascaded converter system. In fact, the small gain theorem is adopted to the linear systems here and applicable to nonlinear systems as well. It can be seen that (6) is merely a sufficient stability criterion because only magnitude information of the interconnected subsystems is needed. Thus, the small gain theorem-based criterion will definitely bring conservatism to the stability analysis and design. The primary benefit of using the small gain theorem is that it determines the so-called MLG for the cascaded converter system, which is easily observed by comparing (5) with (6) and is given by

$$T_{MLG}=Y_{iL}\cdot Z_{OS} \tag{7}$$

where the T_{MLG} is the minor loop gain of the heterogeneous cascaded converter system in Fig. 2(a).

And once the MLG has been defined, the Nyquist-based stability criteria, e.g. GMPM criterion [6] and ESAC criterion [4] with lower conservatism can be utilized to analyze and optimize the heterogeneous cascaded converter system.

Similarly, the stability criterion for the interconnected feedback system model in Fig. 4(b) that corresponding to the other heterogeneous cascaded converter system in Fig. 2(b) is given by

$$\begin{cases}\left|Z_{iL}\cdot Y_{OS}\right|<1\\ T_{MLG}=Z_{iL}\cdot Y_{OS}\end{cases} \tag{8}$$

Observing (7) and (8), one finds that the minor loop gains for the heterogeneous cascaded systems of Fig. 2(a) and (b) are reciprocals to each other. This statement can also be found in the existing published works [8, 9].

In homogeneous cascaded converters systems as Fig. 3, however, the stability criteria can not be derived based on small gain theorem due to non-uniqueness of the feedback interconnections as described in the Section II. For the same homogeneous cascaded converter system in Fig. 3 (b), application of small gain theorem to the possible two feedback interconnection models in Fig. 5 produces totally different MLG and stability results. The straightforward way to investigate the stability of homogeneous cascaded converter systems is from its characteristic equations.

As the heterogeneous cascaded converter system in Fig. 3 (a) where both source and load converters are of voltage types, the bus current and voltage are given by

$$\begin{cases}\hat{i}_{Bus}=\dfrac{\hat{v}_S-\hat{v}_L}{Z_{OS}+Z_{iL}}\\ \hat{v}_{Bus}=\dfrac{Z_{iL}\hat{v}_S+Z_{OS}\hat{v}_L}{Z_{OS}+Z_{iL}}\end{cases} \tag{9}$$

Thus, the bus current and voltage stabilities depends on the characteristic equation which given by

$$D_{Bus}(s)=Z_{OS}(s)+Z_{iL}(s)=0 \tag{10}$$

The stability of the homogeneous cascaded converter system in Fig. 3(a) is guaranteed if (10) has no right half plane zeros. To facilitate measurement, minor change can be made on (9) as

$$D_{Bus}(s)=Z_{OS}(s)Z_{iL}(s)\frac{1}{Z_{Bus}(s)}=0 \tag{11}$$

where Z_{Bus} is the bus impedance that is the parallel combination of Z_{OS} and Z_{iL}, and can be easily measured by injecting current perturbation signal about the steady state operating point of the bus current.

By duality principle, similar analysis can be conducted on the other homogeneous cascaded converter system for Fig. 3 (b) where the characteristic equation is expressed as

$$D_{Bus}(s)=Y_{OS}(s)+Y_{iL}(s)=0 \tag{12}$$

The stability of the homogeneous cascaded converter system in Fig. 3(b) is guaranteed if (10) has no right half plane zeros. And the measurement of frequency-domain response can be achieved via injecting voltage perturbation signal about the steady state operating point of the bus voltage.

Therefore, for the homogeneous cascaded converter systems in Fig. 3(a) and (b), the characteristic equations as (10) and (12) would be directly utilized to judge the stabilities of the cascaded systems to avoid ambiguity in deriving minor loop gains.

IV. Experimental Results

To validate the qualitative analysis, the laboratory prototype of the cascaded Buck DC-DC converter system is built according to the schematic of Fig. 6 where both the source converter and load converter are of voltage mode control with type III compensation networks. The key parameters of the prototype are listed in Table 1 and the photo of the laboratory prototype is shown in Fig. 7.

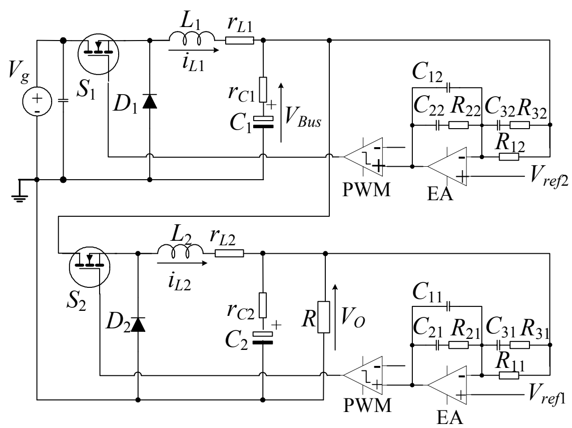


Fig. 6 Schematic of the heterogeneous cascaded converter system with voltage type source converter

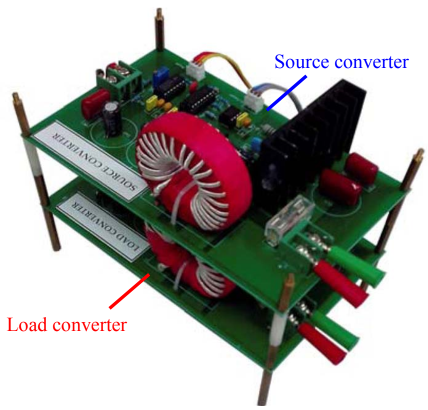


Fig. 7 Prototype of the cascaded converter system

Table 1 Parameters of the cascaded DC-DC converter

Parameter	Value	Parameter	Value
V_g (V)	42	r_{C1} (mΩ)	0.1
V_{Bus} (V)	24	L_2 (μH)	100
V_O (V)	14	C_2 (μF)	47
L_1(μH)	180	r_{L2} (mΩ)	0.12
C_1 (μF)	20	r_{C2} (mΩ)	0.09
r_{L1} (Ω)	0.16	R (Ω)	5
R_{12} (kΩ)	3.3	R_{11} (kΩ)	5.1
R_{22} (Ω)	820	R_{21} (Ω)	330
R_{32} (Ω)	82	R_{31} (Ω)	300
C_{12} (nF)	4.7	C_{11} (nF)	10
C_{22} (nF)	168	C_{21} (nF)	180
C_{32} (nF)	33	C_{31} (nF)	10

According to its operation principle, the feedback interconnection of such heterogeneous cascaded converter systems falls into the category of Fig. 4(a), where the output of source converter and input of load converter are of voltage type and current type respectively. Thus, the minor loop gain defined in (7) is applied to evaluate the relative stability of the cascaded system by GMPM criterion. The Bode plots of the cascaded converter system is shown in Fig. 8 where not only the output impedance of the source converter Z_{OS} and the input impedance of the load converter Z_{iL} but also the minor loop gain of the overall cascaded system T_{MLG} are given. The sufficient phase margins of the minor loop gain says that the cascaded system is sufficiently stable even though there are any magnitude overlap (i.e. loading effect on the source converter) between the output impedance of the source converter and the input impedance of the load converter. The Nyquist diagram of the minor loop gain is also provided with Fig. 9, from which the trajectory does not encircle the (-1, 0) point and the same stability margins can also be easily identified as those of the Bode plots. The steady state operation results are shown in Fig. 10 where the cascaded converter system works well at nominal load. In addition, the transient responses to input voltage pulse and load current pulse are also provided with Fig. 11(a) where the input voltage pulsed between 32 V and 52 V with 10 ms period and Fig. 11(b) where the load current pulsed between 1 A and 6 A with 10 ms period respectively, and the time-domain responses show that the cascaded converter system works stably and agree well with the minor loop gain prediction.

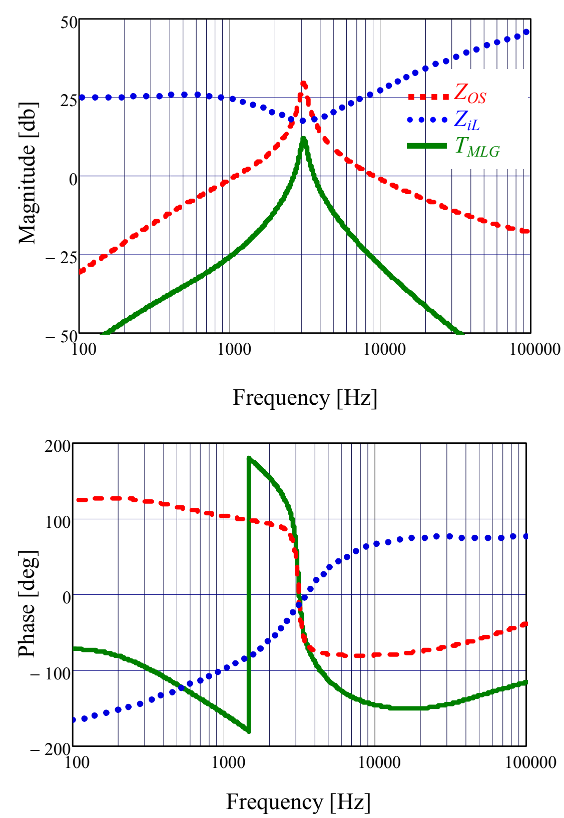


Fig. 8 Bode plots of the cascaded converter system

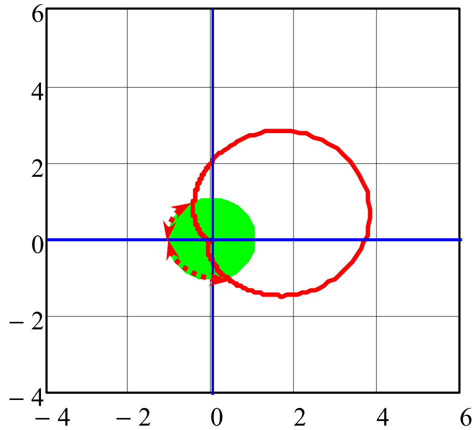


Fig. 9 Nyquist diagram of the minor loop gain

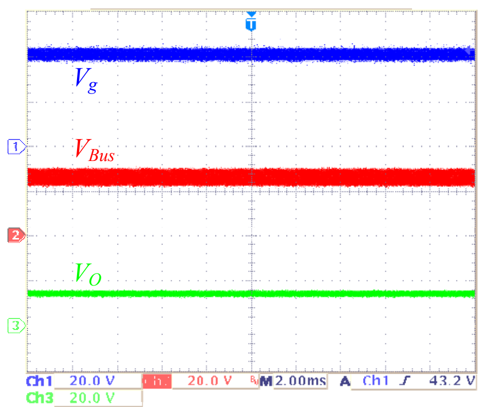
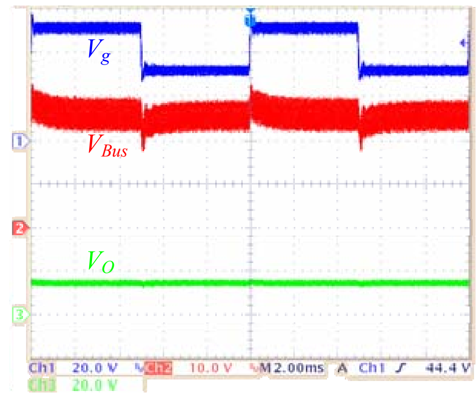
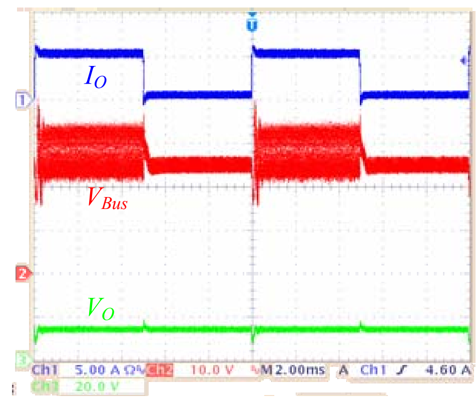


Fig. 10 Steady state operation



(a) Transient response to Input voltage pulse



(b) Transient response to load current pulse

Fig. 11 Transient responses of the bus voltage

V. Conclusion

This work has analyzed feedback interconnections of general cascaded DC-DC converters systems in qualitative manner. It has been shown that small gain theorem-based stability criteria are uniquely determined for heterogeneous cascaded systems, where the minor loop gain concept can be directly applied to analyze the systems by Nyquist plots. But in homogeneous cascaded systems, the feedback interconnections are non-unique, where the stabilities should be evaluated via the characteristic equations to avoid ambiguities in deriving the minor loop gains.

Furthermore, the small gain theorem is not only suitable for linear systems but also suitable for nonlinear systems, which suggests that it can be utilized to large-signal analysis and design of cascaded converter systems in feedback interconnections point of view.

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